Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International Advanced Level

Centre Number	Candidate Number

Specimen Paper ■ : explanation ∵ is 'because' ∴ is 'therefore'

(Time: 1 hour 30 minutes) Paper Reference **WMA11/01**

Mathematics

International Advanced Subsidiary/Advanced Level

Pure Mathematics P1

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

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Pearson

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Answer ALL questions. Write your answers in the spaces provided.

1. The point $P(3, -4)$ lies on the curve with equation $y = \mathrm{f}(x)$

State the coordinates of the image of P under the transformation represented by the curve with equation

(a) $y = \mathrm{f}(x - 2)$ **(1)**

(b) $y = -\mathrm{f}(x)$ **(1)**

(c) $2y = \mathrm{f}(x)$ **(1)**

(d) $y = \mathrm{f}(x) + 4$ **(1)**

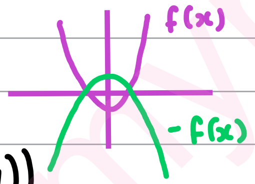
a) $y = f(x-2)$ This means translation $\begin{pmatrix}2\\0\end{pmatrix}$, move graph $y = f(x)$ 2 units to the right. As the -2 is inside $f(x)$ brackets, only x-coordinates are affected & we do inverse of what is in brackets (for (x, y) when $y = f(x-a)$, we have $(x+a, y)$)

$P(3+2, -4)$

$\therefore P(5, -4)$

b) $y = -f(x)$ means reflection of $y = f(x)$ by x-axis.

e.g.



x-coordinates are unaffected.

$P(3, -(-4))$

$\therefore P(3, 4)$

c) $2y = f(x) \longrightarrow y = \frac{1}{2}f(x)$ This means vertical squash by 2, we multiply all y-coordinates of $f(x)$ by $\frac{1}{2}$. x-coordinates are unaffected $\because \frac{1}{2}$ is outside $f(x)$ brackets.

$P(3, \frac{1}{2}(-4))$

$\therefore P(3, -2)$

d) $y = f(x) + 4$ This means translation $\begin{pmatrix}0\\4\end{pmatrix}$, move up by 4 units. x-coordinates are unaffected $\because +4$ is outside $f(x)$ brackets.

$P(3, -4+4)$ $\therefore P(3, 0)$

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Question 1 continued

$\therefore P(3, 0)$

(Total for Question 1 is 4 marks)

Q1

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2. **In this question you must show all steps of your working.**
Solutions relying on calculator technology are not acceptable.

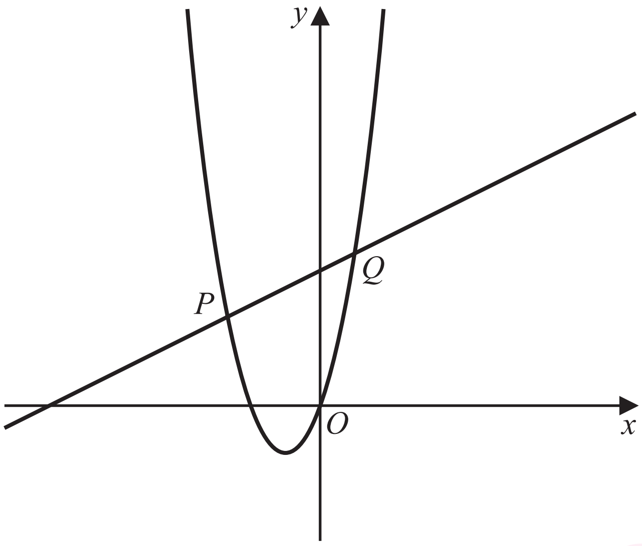

Figure 1

Figure 1 shows a sketch of the curve with equation $y = 2x^2 + 3x$ and the straight line with equation $y = \frac{1}{2}x + 3$

The line meets the curve at the points P and Q, as shown in Figure 1.

(a) Using algebra, find the coordinates of P and the coordinates of Q.
(5)

(b) Hence write down the range of values of x for which $2x^2 + 3x \geqslant \frac{1}{2}x + 3$
(2)

a) ① As line & curve meet, equate both of their equations together.

$$\frac{1}{2}x + 3 = 2x^2 + 3x$$

② rearrange to form quadratic equation.

$$\frac{1}{2}x + 3 = 2x^2 + 3x$$

$-\frac{1}{2}x$: $3 = 2x^2 + \frac{5}{2}x$

-3: $0 = 2x^2 + \frac{5}{2}x - 3$

$\times 2$: $0 = 4x^2 + 5x - 6$

③ Factorise & Solve for x.

$$4x^2 + 5x - 6 = 0$$

Factorise: $(4x-3)(x+2)=0$

Solve: • $4x - 3 = 0$ • $x + 2 = 0$

$4x = 3$ $\therefore x = -2$

$\therefore x = \frac{3}{4}$

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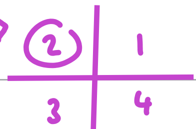
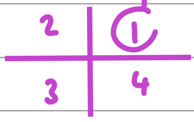
Question 2 continued

P is in 2^{nd} quadrant  $\therefore x < 0$ (negative x) $\therefore x = -2$

Q is in 1^{st} quadrat  $\therefore x > 0$ (positive x) $\therefore x = \frac{3}{4}$

④ Find y-coordinates of P & Q by substituting $x_P = -2$ & $x_Q = \frac{3}{4}$ into equation of curve or line.

$$y = \frac{1}{2}x + 3$$

y-coordinate of P: $y = \frac{1}{2}(-2) + 3 = -1 + 3 = 2$

y-coordinate of Q: $y = \frac{1}{2}(\frac{3}{4}) + 3 = \frac{3}{8} + 3 = \frac{27}{8}$

$\therefore P(-2, 2)$ & $Q(\frac{3}{4}, \frac{27}{8})$

b) $$2x^2 + 3x \geqslant \frac{1}{2}x + 3$$

① rearrange into 1 quadratic equation

$4x^2 + 5x - 6 \geqslant 0$ (from part (a))

② When $4x^2 + 5x - 6 = 0$

$x = -2, \frac{3}{4}$ (from part a)

③ To find inequalities:

1 more than $x = -2$ is $x = -1$. Substitute into quadratic equation & check if inequality is true or false.

$4(-1)^2 + 5(-1) - 6 \geqslant 0$

$-7 \geqslant 0$

This is FALSE $\therefore x$ CANNOT be greater than -2. $\therefore x \leqslant -2$

1 less than $x = \frac{3}{4}$ is $x = -\frac{1}{4}$. Substitute into quadratic equation & check if inequality is true or false.

$4(-\frac{1}{4})^2 + 5(-\frac{1}{4}) - 6 \geqslant 0$

$-7 \geqslant 0$

This is FALSE $\therefore x$ CANNOT be less than $\frac{3}{4}$. $\therefore x \geqslant \frac{3}{4}$

$\therefore x \leqslant -2$ & $x \geqslant \frac{3}{4}$

Q2

(Total for Question 2 is 7 marks)

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3. A curve has equation

$$y = \frac{x^3}{16} - 4\sqrt{x} + \frac{8}{x}, \qquad x > 0$$

(a) Find $\frac{dy}{dx}$ giving your answer in its simplest form.

(3)

The point $P(4, -2)$ lies on the curve.

(b) Use the answer to part (a) to find the equation of the normal to the curve at P, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

a) ① rewrite equation into simpler form for differentiation.

$y = \frac{x^3}{16} - 4\sqrt{x} + \frac{8}{x} = \frac{1}{16}x^3 - 4\sqrt{x} + \frac{8}{x}$

$y = \frac{1}{16}x^3 - 4\sqrt[2]{x^1} + \frac{8}{x} = \frac{1}{16}x^3 - 4x^{\frac{1}{2}} + \frac{8}{x}$

① indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$y = \frac{1}{16}x^3 - 4x^{\frac{1}{2}} + \frac{8}{x^1} = \frac{1}{16}x^3 - 4x^{1/2} + 8x^{-1}$ ②

② indicies rule: $\frac{a}{x^b} = ax^{-b}$

② Differentiate

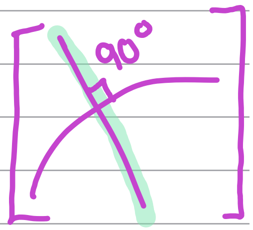
$\frac{dy}{dx} = 3\left(\frac{1}{16}x^{3-1}\right) + \frac{1}{2}\left(-4x^{\frac{1}{2}-1}\right) + (-1)(8x^{-1-1})$

$= \frac{3}{16}x^2 - 2x^{-1/2} - 8x^{-2}$

$\therefore \frac{dy}{dx} = \frac{3}{16}x^2 - 2x^{-1/2} - 8x^{-2}$

↳ can write as $-\frac{8}{x^2}$

b) normal is perpendicular to curve



∴ we find gradient of normal (m_n)

using perpendicular gradient rule $m_{normal} \times m_{curve} = -1$

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Question 3 continued

① Find gradient of curve at point $P(4,-2)$

$$\left.\frac{dy}{dx}\right|_{x=4} = \frac{3}{16}(4)^2 - 2(4)^{-1/2} - 8(4)^{-2} = \frac{3}{2}$$

↳ from part (a)

② Find gradient of normal using formula.

$$M_n \times M_c = -1$$

$$M_n \times 3/2 = -1 \quad \div \tfrac{3}{2}$$

$$M_n = -2/3$$

③ Find equation of normal using line passing through (a, b) & gradient M.

equation: $(y-b) = M(x-a)$

$a = 4$

$b = -2$

$M = -2/3$

$$(y-(-2)) = -2/3(x-4)$$

④ Write equation in form $ax + by + c = 0$

$$y + 2 = -2/3(x-4) \quad \times 3$$

$$3y + 6 = -2(x-4)$$

$$3y + 6 = -2x + 8$$

$$3y + 6 + 2x = 8 \quad +2x$$

$$3y - 2 + 2x = 0 \quad -8$$

$\therefore 2x + 3y - 2 = 0$ $\quad a = 2 \quad b = 3 \quad c = -2$

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Q3

(Total for Question 3 is 7 marks)

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4. **In this question you must show all steps of your working.**
Solutions relying on calculator technology are not acceptable.

(i) Given

$$\frac{4^a}{2^{3b}} = 32\sqrt{2}$$

use the laws of indices to write a in terms of b.

(4)

(ii) Solve the equation

$$3x = x\sqrt{2} + 14$$

giving your answer as a simplified surd.

(3)

i) ① rewrite $\frac{4^a}{2^{3b}}$ in terms of powers of 2.

$$\frac{4^a}{2^{3b}} = \frac{(2^2)^a}{2^{3b}} = \frac{2^{2a}}{2^{3b}} = \frac{2^{2a}}{2^{3b}} = 2^{2a-3b}$$

① indicies rule: $a^{bc} = (a^b)^c = (a^c)^b$ ② indicies rule: $\frac{a^b}{a^c} = a^{b-c}$

② rewrite $32\sqrt{2}$ in terms of powers of 2.

$$32\sqrt{2} = 2^5\sqrt{2} = 2^5 \times 2^{\frac{1}{2}} = 2^5 \times 2^{1/2} = 2^{5+\frac{1}{2}} = 2^{\frac{11}{2}}$$

③ indicies rule: $\sqrt[c]{a^b} = a^{\frac{b}{c}}$ ④ indicies rule: $a^b \times a^c = a^{b+c}$

③ equate the two powers of 2 formed to each other.

$$2^{2a-3b} = 2^{\frac{11}{2}}$$

④ form equation of a in terms of b.

$$2a - 3b = \frac{11}{2}$$

$$2a - 3b = \frac{11}{2}$$

$+3b$ $$2a = \frac{11}{2} + 3b$$ $+3b$

$\div 2$ $$a = \frac{11}{4} + \frac{3b}{2}$$ $\div 2$

$$a = 2.75 + 1.5b$$

$$\therefore a = 1.5b + 2.75$$

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Question 4 continued

ii) ① rearrange to make x the subject.

$$3x = x\sqrt{2} + 14$$
$-x\sqrt{2}$ $\qquad$ $-x\sqrt{2}$
$$3x - x\sqrt{2} = 14$$
$$x(3-\sqrt{2}) = 14$$
$\div(3-\sqrt{2})$ $\qquad$ $\div(3-\sqrt{2})$
$$x = \frac{14}{3-\sqrt{2}}$$

② rationalise denominator & simplify:

$$x = \frac{14}{3-\sqrt{2}} \times \frac{(3+\sqrt{2})}{(3+\sqrt{2})} = \frac{14(3+\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{14(3+\sqrt{2})}{9+3\sqrt{2}-3\sqrt{2}-2} = \frac{14(3+\sqrt{2})}{7}$$

$$= \frac{7(2(3+\sqrt{2}))}{7} = 2(3+\sqrt{2}) = 6+2\sqrt{2}$$

$$\therefore x = 6+2\sqrt{2}$$

markscheme accepts form $2(3+\sqrt{2})$

Q4

(Total for Question 4 is 7 marks)

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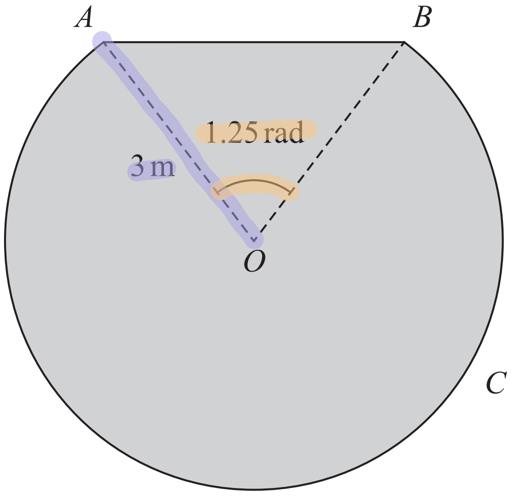


Figure 2

Figure 2 shows the plan view of a design for a garden pond.

The pond consists of a sector, $AOBCA$, of a circle with centre O, joined to a triangle AOB.

Given $AO = BO = 3$ m and angle $AOB = 1.25$ radians,

↳ radius ↳ units!

(a) find the perimeter of the pond, giving your answer, in metres, to 2 decimal places. **(4)**

↙ all of pond is the same depth

Given that there is a uniform depth of water in the pond of 1.5 m,

(b) find the volume of water in the pond, in m³, to one decimal place. **(4)**

a) Perimeter = arc BCA + Side AB.

① find length of arc BCA. length of arc: $S = r\theta$

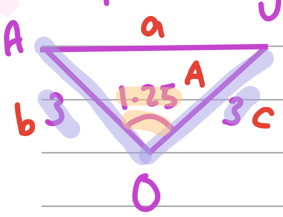
$r = 3$ m

$\theta = 2\pi - 1.25$

↳ angle in a circle is 2π (360°)

$S = 3(2\pi - 1.25) = 15.09955... \approx 15.100$ (3dp)

② Find length of side AB using Cosine rule



$AB^2 = OA^2 + OB^2 + 2(OA)(OB)\cos \angle AOB$

Pure Mathematics P1

Mensuration

Surface area of sphere $= 4\pi r^2$

Area of curved surface of cone $= \pi r \times$ slant height

Cosine rule

$a^2 = b^2 + c^2 - 2bc\cos A$

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Question 5 continued

$AB^2 = 3^2 + 3^2 + 2(3)(3)\cos(1.25)$

$AB^2 = 12.32419...$

$AB = \sqrt{12.3241...} = 3.510583... \approx \underline{3.511}$ (3dp)

③ Find Total Perimeter:

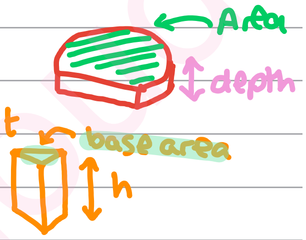
Perimeter = BCA + AB = 15.100 + 3.511 = 18 611

≈ 18.61

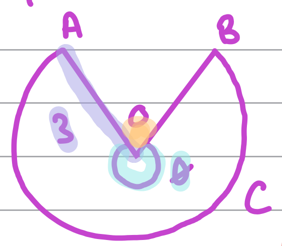
∴ Perimeter of pond = 18.61m (2dp)

b) Volume = Area of pond × depth

↳ same as volume of prism: V = base area × height



① Find Area of Sector AOBCA



Area of Sector: $A = \frac{1}{2}r^2\theta$

radius = 3m

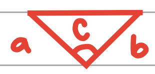
$\theta = (2\pi - 1.25)$ from part (a)

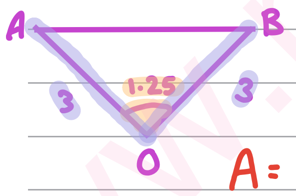
$A = \frac{1}{2} \times 3^2 \times (2\pi - 1.25) = 22.64933...$

$\approx \underline{22.659}$ (3dp)

② Find Area of triangle AOB.

Area of a triangle: $A = \frac{1}{2}ab\sin C$





a = OA = 3

b = OB = 3

C = ∠AOB = 1.25

$A = \frac{1}{2} \times 3 \times 3 \times \sin 1.25 = 4.2704307... \approx \underline{4.270}$ (3dp)

③ Total Area = ① + ② = 22.659 + 4.270 = $\underline{26.929}$

④ Volume = Area × depth = 26.929 × 1.5 = 40.3935 ≈ 40.4

∴ Volume = $40.4m^3$ (1dp)

(Total for Question 5 is 8 marks)

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Q5

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6.

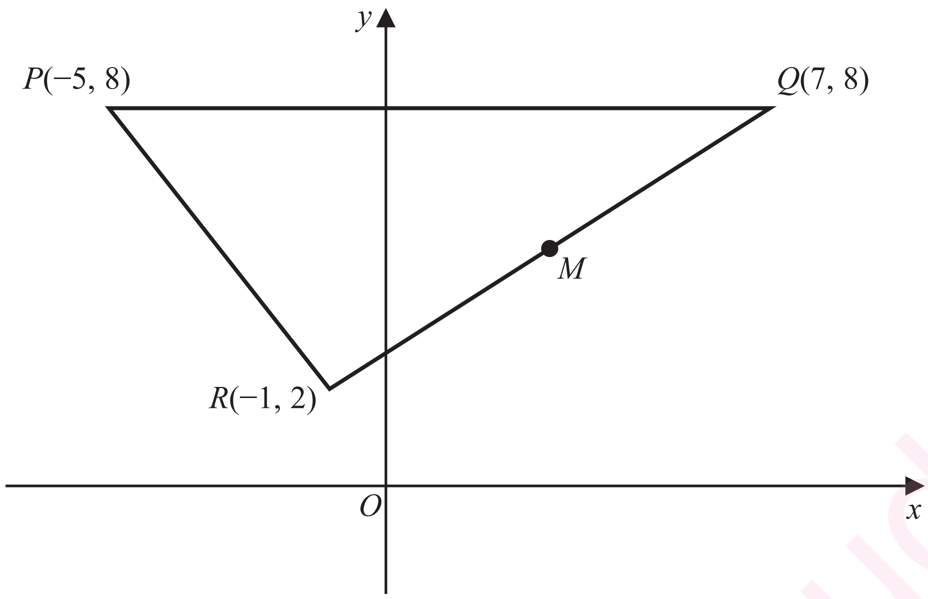

Figure 3

The points $P(-5, 8)$, $Q(7, 8)$ and $R(-1, 2)$ form the vertices of a triangle PQR, as shown in Figure 3. The point M is the midpoint of QR.

The line l passes through M and is parallel to PR.

(a) Find an equation for l, writing your answer in the form $ax + by + c = 0$, where a, b and c are integers to be found.

(4)

The line l cuts PQ at the point N.

(b) Find

(i) the coordinates of N,

(ii) the area of triangle MNQ.

(3)

a) l is parallel to PR ∴ has same gradient as line PR.

① find gradient of line PR. gradient formula $m = \frac{y_1 - y_2}{x_1 - x_2}$

P(−5, 8)

R(−1, 2) $m = \frac{8-2}{-5-(-1)} = \frac{6}{-4} = -\frac{3}{2}$ = gradient of line l

② find coordinates of point M.

M is midpoint of QR. midpoint : $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

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Question 6 continued

$Q(7,8)$
$R(-1,2)$

$M\left(\frac{7+(-1)}{2}, \frac{8+2}{2}\right) = \left(\frac{6}{2}, \frac{10}{2}\right) = (3,5)$

$\therefore M(3,5)$

③ Find equation of line l using line passing through (a, b) & gradient M.

equation: $(y-b) = M(x-a)$

$M(3,5)$
$a = 3$
$b = 5$
$M = -3/2$

$(y-5) = -3/2\,(x-3)$

④ rewrite equation in the form $ax+by+c=0$

$y-5 = -3/2\,(x-3)$

×2 $2y-10 = -3(x-3)$ ×2

$2y-10 = -3x+9$

+3x $2y+3x-10 = 9$ +3x

−9 $2y+3x-19 = 0$ −9

$\therefore 3x+2y-19=0$ $a=2$ $b=3$ $c=-19$

b) i) ① first we need to find equation of line PQ.

PQ is a horizontal line parallel to x-axis ∴ gradient = 0.

or: gradient formula $M = \frac{y_1-y_2}{x_1-x_2} = \frac{8-8}{-5-7} = \frac{0}{-12} = 0$

$(y-b) = M(x-a)$

$y-8 = 0(x-(-5))$

$y-8 = 0$ $\therefore PQ: y = 8$ ← y-coordinate of N

② Substitute $y=8$ into equation l & Solve for x.

$l: 3x+2y-19=0$

$3x+2(8)-19=0 \rightarrow 3x+16-19=0 \rightarrow 3x-3=0$

Q6

(Total for Question 6 is 7 marks)

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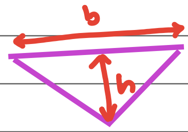
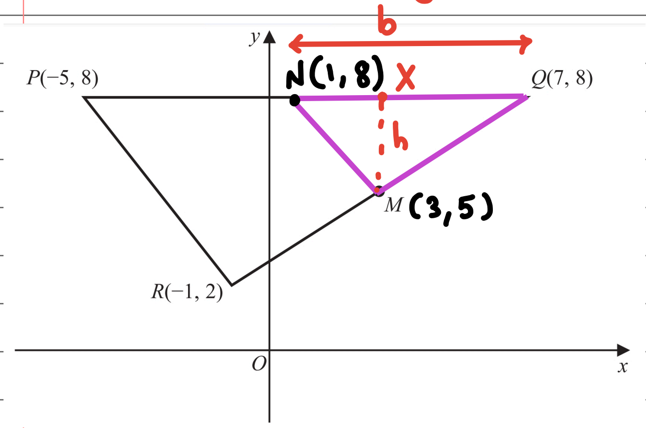
$3x = 3$

$\therefore x = 1$ ← x-coordinate of N

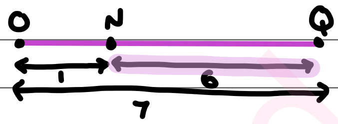
$\therefore N(1, 8)$

ii) Area of a triangle : $A = \frac{1}{2}bh$





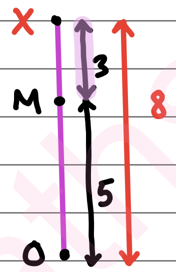
base : $NQ = 7 - 1 = 6$



OR use : length between 2 points : $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\sqrt{(7-1)^2 + (8-8)^2} = \sqrt{6^2 + 0^2} = \sqrt{6^2} = 6$

height : $MX = 8 - 5 = 3$



$\therefore A = \frac{1}{2} \times 6 \times 3 = \frac{18}{2} = 9$

$\therefore$ Area of triangle MNQ = 9 units^2

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7. (a) Sketch, on Diagram 1, the graphs of

units is degrees

(i) $y = 2\cos x, \quad -90° \leqslant x \leqslant 360°$

(ii) $y = \tan x, \quad -90° \leqslant x \leqslant 360°$

(4)

(b) Given that $n \in \mathbb{N}$, deduce, in terms of n, the number of real solutions of the equation

(i) $2\cos x = \tan x, \quad -90° \leqslant x \leqslant (360n)°$

(ii) $\tan x = -\frac{3}{2}, \quad -90° \leqslant x \leqslant (360n)°$

(2)

a)

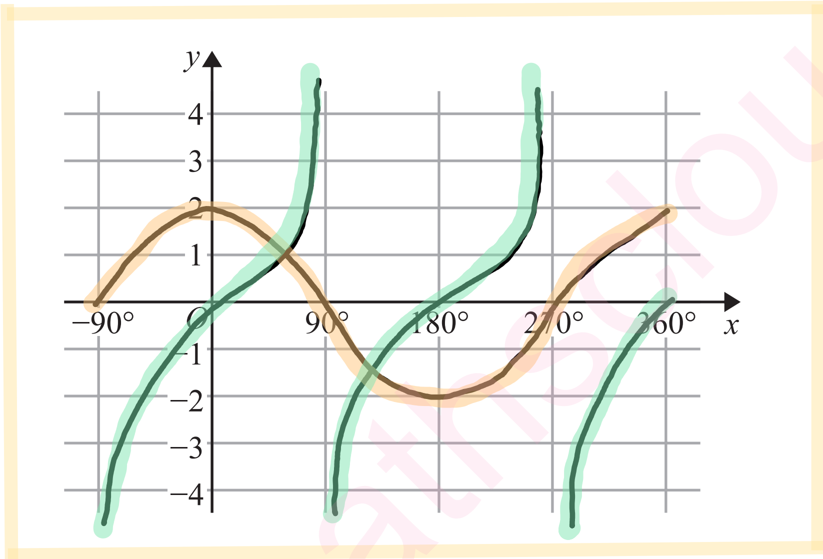
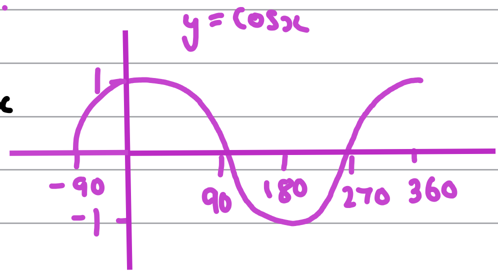


Diagram 1

(If you make an error there is a spare copy of Diagram 1 on the next page.)

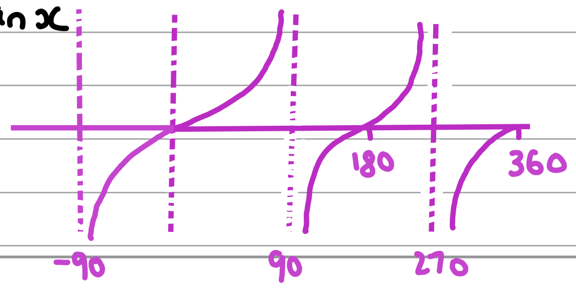
a) i) working out : $y = 2\cos x$ is same as $y = 2f(x)$. This means vertical stretch by 2. So all y-coordinates of $y = \cos x$ are multiplied by 2. x-coordinates unchanged ∵ 2 is outside f(x) brackets.

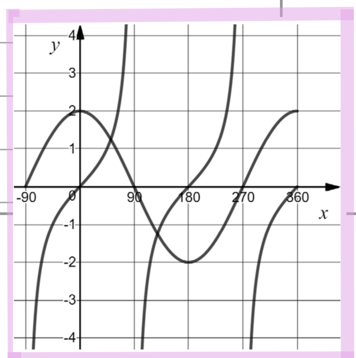
$y = \cos x$



Markscheme :

ii) $y = \tan x$





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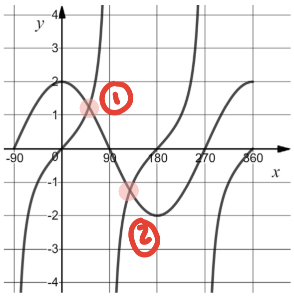
Question 7 continued

b) i) $2\cos x = \tan x$

in region $0° \leq x \leq 360°$



2 intersections ∴ 2 solutions.

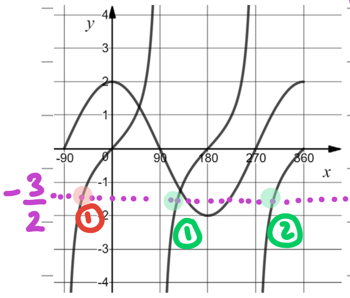
in region $-90 \leq x \leq (360n)$

To get from 360 to $(360n)$, multiply by n

number of solutions $= 2 \times n$

∴ $2n$

ii) $\tan x = -\frac{3}{2}$



in region $0° < x < 360°$, for every period of 360°

2 intersections ∴ 2 solutions.

in region $-90 \leq x \leq (360n)$

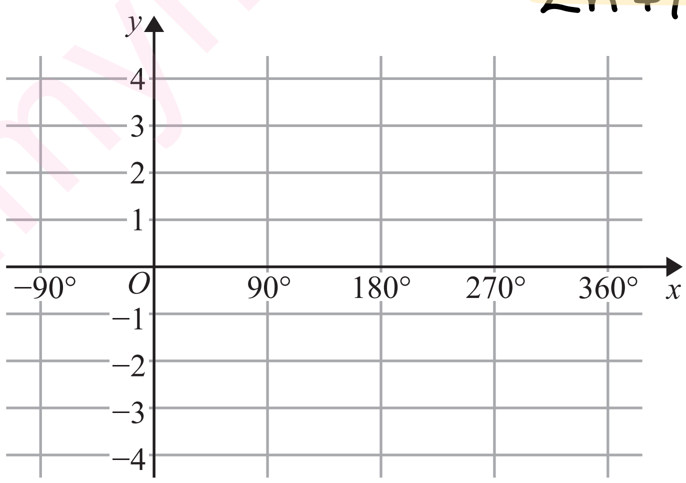
To get from 360 to $(360n)$, multiply by n

number of solutions $= 2 \times n$

INCLUDE 1 solution from region $-90° \leq x \leq 0°$

$2n + 1$

∴ $2n + 1$



Spare copy of Diagram 1

(Only use this diagram if you have made an error on Diagram 1.)

Q7

(Total for Question 7 is 6 marks)

Leave blank

8.

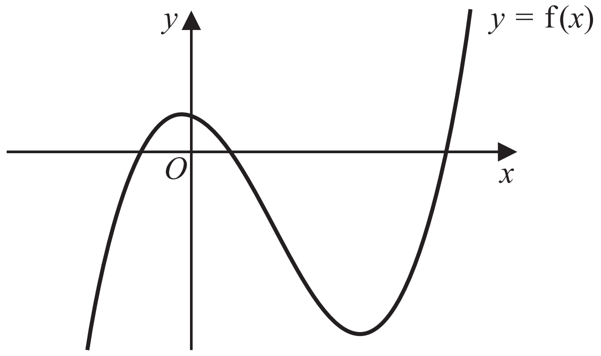


Figure 4

Figure 4 shows a sketch of a curve with equation $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = (x + 2)(x - 10)(2x - 3)$$

(a) Deduce the values of x for which

$$\mathrm{f}(x) > 0$$

(2)

(b) Expand $\mathrm{f}(x)$ to the form

$$2x^3 + ax^2 + bx + 60$$

where a and b are integers to be found.

(3)

A copy of Figure 4, called Diagram 2, is shown on the next page.

(c) (i) Sketch, on Diagram 2, the curve with equation $y = \frac{k}{x} + 60$, where k is a positive constant. You must show clearly the position and equation of the horizontal asymptote to the curve.

(ii) Hence deduce the number of real roots of the equation

$$\mathrm{f}(x) = \frac{k}{x} + 60 \quad \text{where } k \text{ is a positive constant}$$

(4)

a) ① find critical values of f(x).
when f(x) = 0
$(x+2)(x-10)(2x-3) = 0$

- $x + 2 = 0$ ∴ $x = -2$
- $x - 10 = 0$ ∴ $x = 10$
- $2x - 3 = 0$, $2x = 3$ ∴ $x = \frac{3}{2}$

② Find inequalities:

$$f(x) > 0$$
$$(x+2)(x-10)(2x-3) > 0$$

→ When x is 1 less than -2. $x = -3$ Substitute into equation & check if inequality is true or false.

$$(-3+2)(-3-10)(2(-3)-3) > 0$$
$$-117 > 0$$

This is FALSE ∴ x cannot be less than -2. ∴ $x > -2$

→ When x is 1 less than 10. $x = 9$ Substitute into equation & check if inequality is true or false.

$$(9+2)(9-10)(2(9)-3) > 0$$
$$-165 > 0$$

This is FALSE ∴ x cannot be less than 10. ∴ $x > 10$

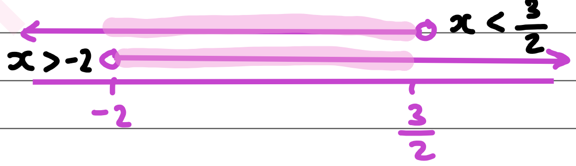
→ When x is 1 more than $\frac{3}{2}$. $x = \frac{5}{2}$ Substitute into equation & check if inequality is true or false.

$$\left(\tfrac{5}{2}+2\right)\left(\tfrac{5}{2}-10\right)\left(2\left(\tfrac{5}{2}\right)-3\right) > 0$$
$$-67.5 > 0$$

This is FALSE ∴ x cannot be more than $\frac{2}{3}$. ∴ $x < \frac{2}{3}$

③ Inequalities are: $x > -2$, $x > 10$, $x < \frac{2}{3}$

$x > -2$ & $x < \frac{3}{2}$ can form one inequality together as they overlap in range:



∴ $-2 < x < \frac{3}{2}$

∴ $-2 < x < \frac{3}{2}$ & $x > 10$

b) Expanding brackets:

$$f(x) = (x+2)(x-10)(2x-3)$$
$$f(x) = (x^2+2x-10x-20)(2x-3) = (x^2-8x-20)(2x-3)$$

$$f(x) = (x^2-8x-20)(2x-3)$$
$$= (2x^3-3x^2-16x^2+24x-40x+60)$$

$$= 2x^3-19x^2-16x+60$$

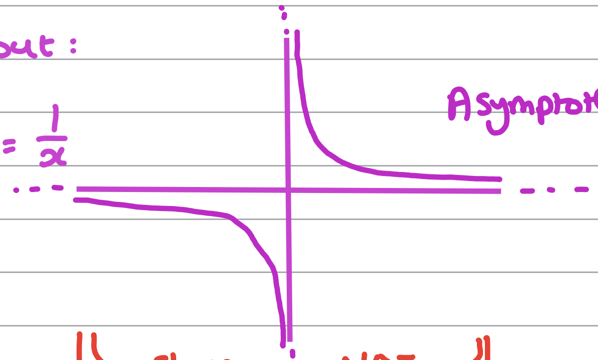
Leave blank

Question 8 continued

$\therefore f(x) = 2x^3 - 19x^2 - 16x + 60$ $a = -19$ $b = -16$

c)i) Working out :

graph of $y = \frac{1}{x}$



Asymptotes : $y = 0$
$x = 0$

K positive ∴ shape NOT

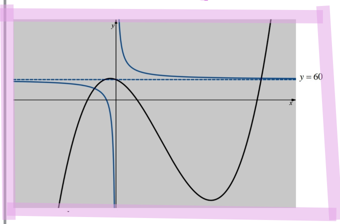
for $y = \frac{K}{x} + 60$ if $g(x) = \frac{1}{x}$, $g(Kx) + 60 = \frac{K}{x} + 60$

Translation : horizontal stretch by K & $\begin{pmatrix} 0 \\ 60 \end{pmatrix}$, move up by 60 units.

vertical asymptote : $x = 0 \times K \rightarrow x = 0$

horizontal asymptote : $y = 0 + 60 \rightarrow y = 60$

Markscheme :



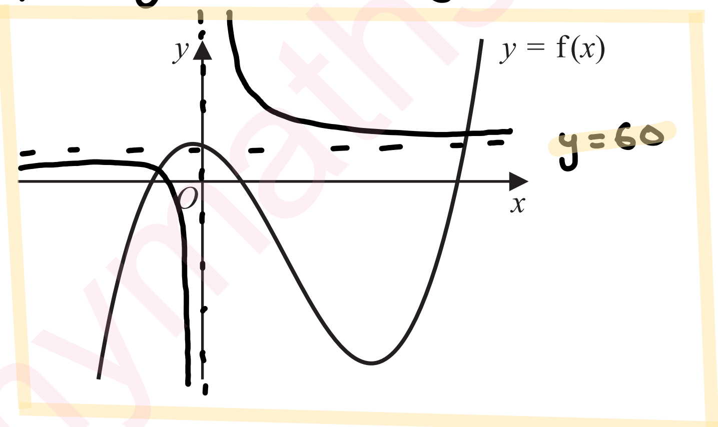
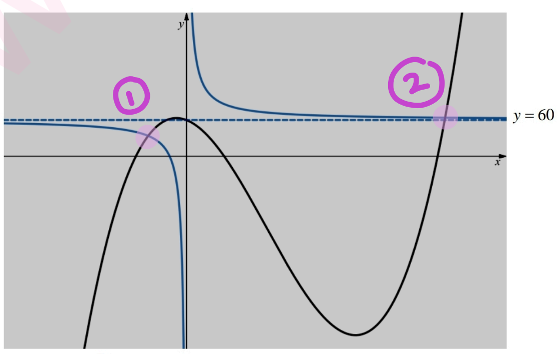


Diagram 2

ii) real roots is number of intersections

$f(x) = \frac{K}{x} + 60$ is $2x^3 - 19x^2 - 16x + 60 = \frac{K}{x} + 60$



2 intersections

∴ 2 real roots

Q8

(Total for Question 8 is 9 marks)

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9.

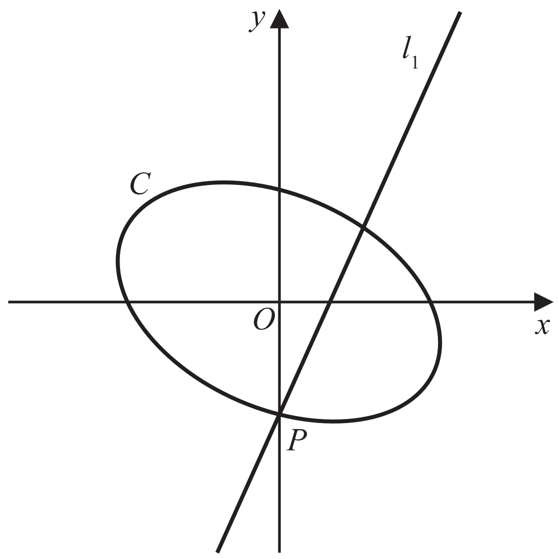

Figure 5

Figure 5 shows a sketch of the curve C with equation

$$4x^2 + 9y^2 + 4xy = 64$$

The curve C crosses the negative y-axis at the point P.

The line l_1 has gradient 2 and passes through P, as shown in Figure 5.

(a) Find the equation of l_1 giving your answer in the form $y = mx + c$, where m and c are constants to be found.

(3)

The line l_2 has equation $y = 2x + k$, where k is a constant.

(b) Show that the y coordinate of any points where l_2 meets C are solutions of the equation

$$12y^2 - 4ky + k^2 - 64 = 0$$

(3)

Given that l_2 meets C at two distinct points,

(c) find the range of possible values for k.

(4)

a) ① find coordinates of P.
P lies on the y-axis $\therefore x=0$.

Substitute $x=0$ into equation C.

$C: 4x^2 + 9y^2 + 4xy = 64$

$4(0)^2 + 9y^2 + 4(0)y = 64$

$9y^2 = 64 \longrightarrow y^2 = \frac{64}{9}$

Leave blank

Question 9 continued

$$y^2 = \frac{64}{9}$$
$$y = \pm\sqrt{\frac{64}{9}}$$
$$y = \pm\frac{8}{3}$$

P is on negative y-axis $\therefore y < 0 \quad \therefore y = -\frac{8}{3}$

$P(0, -8/3)$

② Find equation of line l using line passing through (a, b) & gradient m.

equation : $(y - b) = m(x - a)$

$a = 0$
$b = -8/3$
$m = 2$

$$(y - (-\tfrac{8}{3})) = 2(x - 0)$$

③ Write equation in form $y = mx + c$

$$y + 8/3 = 2(x)$$
$$y + 8/3 = 2x \quad -8/3$$
$$y = 2x - 8/3$$

Alternative method: gradient $(m) = 2$ & y-intercept $(c) = -8/3$
$\therefore$ Substitute into $y = mx + c$

$$\therefore y = 2x - 8/3$$

b) Substitute $y = 2x + k$ of l_2 into equation C.

$$y = 2x + k \longrightarrow x = \tfrac{1}{2}(y - k)$$

$$C : 4x^2 + 9y^2 + 4xy = 64$$
$$4\left(\tfrac{1}{2}(y-k)\right)^2 + 9y^2 + 4y\left(\tfrac{1}{2}(y-k)\right) = 64$$
$$4\left(\tfrac{1}{4}(y^2 - 2ky + k^2)\right) + 9y^2 + 2y(y-k) = 64$$
$$4\left(\tfrac{1}{4}(y^2 - 2ky + k^2)\right) + 9y^2 + 2y(y-k) = 64$$
$$(y^2 - 2ky + k^2) + 9y^2 + 2y^2 - 2ky = 64$$
$$y^2 - 2ky + k^2 + 9y^2 + 2y^2 - 2ky = 64$$

Turn over

S 6 3 0 1 0 A 0 1 9 2 4

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Question 9 continued

$$12y^2 - 4ky + k^2 = 64$$

-64 $\quad$ -64

$$12y^2 - 4ky + k^2 - 64 = 0$$

$$\therefore\ 12y^2 - 4ky + k^2 - 64 = 0$$

c) meet at 2 distinct points, so 2 real solutions $\therefore$ use discriminant rule $b^2 - 4ac > 0$

two real roots means that when the equation of the graph is $ay^2 + by + c = 0$ discriminant is $b^2 - 4ac > 0$

(a) (b) (c)

$$12y^2 - 4ky + k^2 - 64 = 0$$

$$b^2 - 4ac > 0$$

$$(-4k)^2 - 4(12)(k^2 - 64) > 0$$

$$16k^2 - 48k^2 + 3072 > 0$$

$$-32k^2 + 3072 > 0$$

Solve to find critical values:

-3072 $\quad$ $-32k^2 + 3072 = 0$ $\quad$ -3072

$\div -32$ $\quad$ $-32k^2 = -3072$ $\quad$ $\div -32$

$$k^2 = 96$$

$$k = \pm\sqrt{96}$$

$$k = \pm\sqrt{16 \times 6} = \pm\sqrt{16}\sqrt{6} = \pm 4\sqrt{6}$$

critical values: $k_1 = -4\sqrt{6}$ $\quad$ $k_2 = +4\sqrt{6}$

$$k_1 < k < k_2$$

where $k_1 < k_2$

$$\therefore\ -4\sqrt{6} < k < 4\sqrt{6}$$

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Question 9 continued

Q9

(Total for Question 9 is 10 marks)

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10. (i) Find $\int \frac{2x-1}{4\sqrt{x}}\,dx$ writing your answer in simplest form.

(5)

(ii) A curve C has equation $y = f(x)$.

Given

- $f'(x) = ax^2 + b$, where a and b are constants
- the gradient of C at the point (3, 5) is 4
- the y intercept of C is -5

find $f(x)$

(5)

i) ① rewrite into form easier for integration.

$$\frac{2x-1}{4\sqrt{x}} = \frac{2x}{4\sqrt{x}} - \frac{1}{4\sqrt{x}} = \left(\frac{2}{4} \times \frac{x}{\sqrt{x}}\right) - \left(\frac{1}{4} \times \frac{1}{\sqrt{x}}\right)$$

$$= \left(\frac{1}{2} \times \frac{x}{\sqrt[2]{x^1}}\right) - \left(\frac{1}{4} \times \frac{1}{\sqrt[2]{x^1}}\right) = \left(\frac{1}{2} \times \frac{x}{x^{1/2}}\right) - \left(\frac{1}{4} \times \frac{1}{x^{1/2}}\right)$$

① indicies rule : $\sqrt[c]{a^b} = a^{\frac{b}{c}}$

$$= \left(\frac{1}{2} \times \frac{x^1}{x^{1/2}}\right) - \left(\frac{1}{4} \times \frac{1}{x^{1/2}}\right) = \left(\frac{1}{2} \times x^{1-1/2}\right) - \left(\frac{1}{4} \times \frac{1}{x^{1/2}}\right)$$

$$= \left(\frac{1}{2} x^{1/2}\right) - \left(\frac{1}{4} \times \frac{1}{x^{1/2}}\right)$$

② indicies rule : $\frac{a^b}{a^c} = a^{b-c}$

$$= \left(\frac{1}{2} x^{1/2}\right) - \left(\frac{1}{4} \times \frac{1}{x^{1/2}}\right) = \left(\frac{1}{2} x^{1/2} - \frac{1}{4} x^{-1/2}\right)$$

③ indicies rule : $\frac{a}{x^b} = ax^{-b}$

② Integrate

$$\int \frac{1}{2}x^{1/2} - \frac{1}{4}x^{-1/2}\,dx = \left[\left(\frac{1/2}{1/2+1} x^{1/2+1}\right) + \left(\frac{-1/4}{-1/2+1} x^{-1/2+1}\right)\right]$$

$$= \frac{1}{3}x^{3/2} - \frac{1}{2}x^{1/2} + C$$

DON'T FORGET or will lose a mark.

$$\therefore \frac{1}{3}x^{3/2} - \frac{1}{2}x^{1/2} + C$$

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Leave blank

Question 10 continued

ii) $f(x) \xrightarrow{\text{differentiate}} f'(x)$, $f'(x) \xrightarrow{\text{integrate}} f(x)$

① at (3,5) gradient of C is 4. Substitute $x=3$ into the gradient function $f'(x)$ (differential is gradient function) so that $f'(3)=5$

$$f'(x) = ax^2 + b$$
$$f'(3) = a(3)^2 + b = 4$$
$$9a + b = 4 \quad (1)$$

② Integrate $f'(x)$ to get $f(x)$.

$$f(x) = \int f'(x)\,dx = \int ax^2 + bx^0\,dx = \left[\left(\frac{a}{2+1}x^{2+1}\right) + \left(\frac{b}{0+1}x^{0+1}\right)\right] = \frac{a}{3}x^3 + bx + c$$

($\because x^0 = 1$)

y-intercept of C is -5, so C passes through $(0,-5)$.
Substitute $(0,-5)$ into $f(x)$, so that $f(0)=-5$, to find $+c$.

$$f(0) = \frac{a}{3}(0)^3 + b(0) + c = -5$$
$$c = -5$$

$$\therefore f(x) = \frac{a}{3}x^3 + bx - 5$$

③ Substitute (3,5) into $f(x)$ so that $f(3)=5$.

$$f(3) = \frac{a}{3}(3)^3 + b(3) - 5 = 5$$
$$9a + 3b - 5 = 5$$
$$9a + 3b = 10 \quad (2)$$

④ Solve simultaneous equations obtained to find values of a & b.

$$(1)\quad 9a + b = 4$$
$$(2)\quad 9a + 3b = 10$$

$$9a + 3b = 10$$
$$9a + b = 4 \quad (-)$$
$$0 + 2b = 6$$
$$\div 2 \quad b = 3 \quad \div 2$$

S 6 3 0 1 0 A 0 2 3 2 4

Question 10 continued

$9a + b = 4$ Substitute $b = 3$ to find a.

$9a + 3 = 4$ (−3)

$9a = 1$ (÷9)

$a = \frac{1}{9}$

$\therefore a = \frac{1}{9} \quad b = 3$

⑤ find $f(x)$.

$f(x) = \frac{a}{3}x^3 + bx - 5$

$a = \frac{1}{9} \quad b = 3$

$f(x) = \frac{1/9}{3}x^3 + 3x - 5$

$\therefore f(x) = \frac{1}{27}x^3 + 3x - 5$

Q10

(Total for Question 10 is 10 marks)

TOTAL FOR PAPER IS 75 MARKS

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